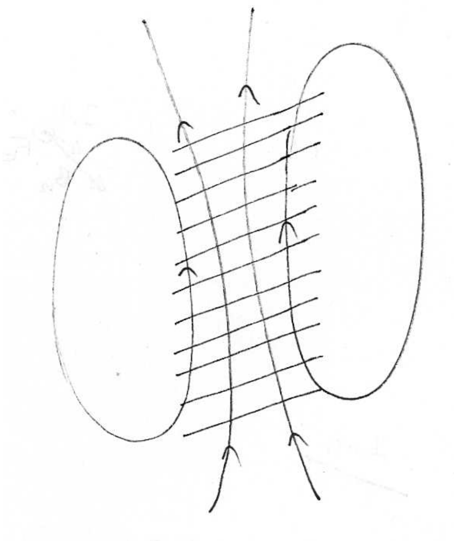
Solutions: Homework 0

Warm-ups

Ex 0.1 Thomson's Jumping Ring

a) Magnetic field



b) As suggested in the question we first split the magnetic field into radial and axial components, so that

$$\boldsymbol{B}(r,z,t) = B_r(r,z)\sin(\omega t)\boldsymbol{e}_r + B_z(r,z)\sin(\omega t)\boldsymbol{e}_z \ . \tag{1}$$

We assume that $B_z(r < a, z > 0)$ is positive: This choice simply amounts to choosing the phase of the time-dependence. It follows that $B_r(r, z > 0)$ must be positive too. By Faraday's law, the emf induced in the ring is

$$\mathcal{E} = -\frac{\omega}{c}\Phi_z(z)\cos(\omega t)\boldsymbol{e}_\theta \ , \tag{2}$$

where Φ_z is the flux through the ring due to the field $B_z(r,z)\boldsymbol{e}_z$. Explicitly $\Phi_z(z) = 2\pi\int_0^a B_z(r,z)dr$.

From eq. (2) we see that the *emf* induced in the ring lags the magnetic field in the solenoid by a phase of $\pi/2$. As mentioned in lecture, if the ring was purely resistive, then the current in the ring would be in phase with the emf, and no net average force would be exerted on the ring. However, if the ring is purely inductive, with inductance L, then the current in the ring is related to the emf via

$$\mathcal{E} = L\frac{dI}{dt} \ , \tag{3}$$

Hence the current in the purely inductive ring

$$I d\boldsymbol{l} = -\frac{a d\theta}{Lc}\Phi_z(z)\sin(\omega t)\boldsymbol{e}_\theta \ . \tag{4}$$

Now, the Lorentz force acting on this current is

$$\begin{aligned} \boldsymbol{F} &= \oint I d\boldsymbol{l} \times \boldsymbol{B}_r \\ &= -\frac{2\pi a}{Lc}\Phi_z(z) B_r(a,z)\sin^2(\omega t)\boldsymbol{e}_\theta \times \boldsymbol{e}_r \\ &= \frac{2\pi a}{Lc}\Phi_z(z) B_r(a,z)\sin^2(\omega t)\boldsymbol{e}_z \end{aligned} \tag{5}$$

which is strictly upwards for $z > 0$. That is, the ring experiences an upwards force if it is located around the upper half of the solenoid.

c) If the ring has both a resistance R and an inductance L, then we must have

$$\mathcal{E} = L\frac{dI}{dt} + IR \ . \tag{6}$$

As a result, the current is shifted by a phase $\pi/2 - \tan^{-1}(\omega L/R)$ relative to the magnetic field.

Ex 0.2: Coaxial Cable

a) i) By Gauss' law, inside the inner cylinder we have $E 2\pi r = 4\pi^2 \rho r^2$, so that

$$\boldsymbol{E} = 2\pi\rho r \boldsymbol{e}_r \ . \tag{7}$$

i) Between cylinders, we have $E 2\pi r = 4\pi^2 \rho a^2$, so that

$$\boldsymbol{E} = 2\pi\rho\frac{a^2}{r}\boldsymbol{e}_r \ . \tag{8}$$

i) Finally, since the entire cable is electrically neutral, then the electric field outside the cable is zero.

b) See sketch below.

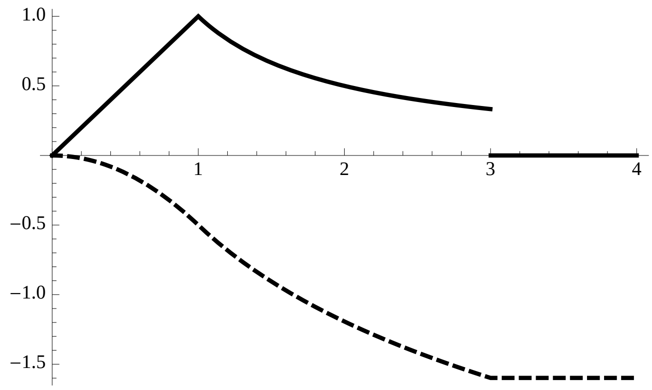
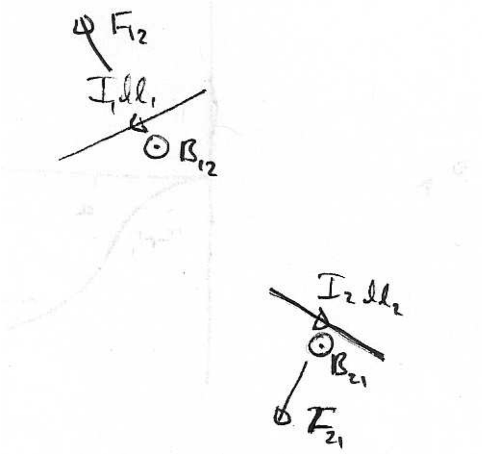


FIG. 1. Electric field strength (bold) and potential (dashed).

Ex 0.3: Newton's Third Law

Suppose $I_1 dl_1$ and $I_2 dl_2$ are coplanar as shown. It is clear that $\boldsymbol{F}_{12} \neq \boldsymbol{F}_{21}$.



The third law is not really violated here, provided we realize that charge conservation requires that an infinitesmal current element - or any open current - can only exist physically if we attach charge reservoirs at each end of the current element: charge is accelerated out of one reservoir, travels through the element, and decelerates in the second reservoir. The key is that - as you will learn a bit later - accelerating charges produce electromagnetic radiation, which carries momentum.