Homework 2

Mostly about Electric and Magnetic Multipoles

Ex 2.1: Particle moving in Magnetic Field

Heald & Marion, ex. 1-30.

Ex 2.2: Higher-order Multipole Moments

a) The definition of the dipole moment contains a reference to the origin of the coordinate system. Show that the electric dipole moment of a system of charges is independent of the choice of origin if the system has zero net charge.

b) If there is a net charge, show that one can choose the origin in such a way that the dipole moment is zero.

c) Extend your observation in (a) to the quadrupole tensor. Show that the electric quadrupole tensor of a system of charges is independent of the choice of the origin if the system has zero net charge and zero electric dipole moment.

d) Can you also extend your observation of (b): If a system of charges has zero net charge, but nonzero dipole moment, can you choose the origin such that the electric quadrupole tensor is zero?

Ex 2.3: Multipole Expansion

A charge $q_1 = 2e$ is located at the origin, and a charge $q_2 = -e$ is located at the point $\mathbf{r} = \mathbf{e}_x$, where $\mathbf{e}_x$ is the unit vector in the x-direction.

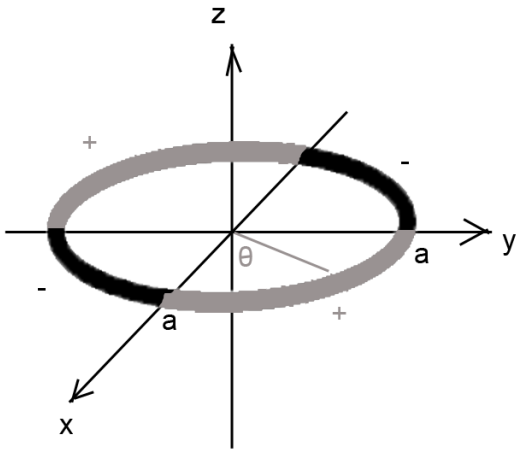
a) Calculate the potential Φ at the positions $\mathbf{r}_1 = r_1\mathbf{e}_x$ and $\mathbf{r}_2 = r_2\mathbf{e}_y$

b) Find the first three moments of the charge distribution.

c) Calculate the potential Φ at $\mathbf{r}_1$ and $\mathbf{r}_2$ using the first three terms of the multipole expansion. Compare your answer to (a) and discuss the difference.

Ex 2.4: Charged ring

Consider a ring of radius a lying in the xy-axis, with line-charge density $\rho_l = \pm\lambda$ (for the grey/black segments respectively, see below) where $\lambda = \text{const}$:



a) Sketch $\rho_l(\theta)$ as a function of the angle θ.

b) Convert the discrete quadrupole formula

$$Q_{ij} = \sum_{\alpha} q_\alpha \left(3x'_{\alpha,i}x'_{\alpha,j} - r'^2_\alpha \delta_{ij}\right)$$

to a form suitable for this continuous charge distribution

c) Derive all elements of the quadrupole tensor Q_{ij}.
Hint: Be sure to use symmetries, and point out which entries are trivially zero. The trig identity $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ *may be useful.*

Ex 2.5: Force on a magnetic dipole

Consider a magnetic dipole with dipole moment m.

a) Show that there is no net force on the dipole if the dipole is placed in a spatially uniform magnetic field B.

b) Two magnetic moments that are brought together exert a force on each other. Explain the origin of this force.

Suggested Heald & Marion problems for further study:

- 2-2: Magnetic field of the earth
- 2-6: Two-dimensional dipole*
- 2-13: Quadrupole moment
- 2-22: Gyromagnetic ratio

* Please disregard the remark on 'Cylindrical symmetry'. Usually, the words 'Cylindrical symmetry' are used for systems that have rotational symmetry around the z axis. In this case, there is translation symmetry in the z direction, but not rotation symmetry.