Homework 5

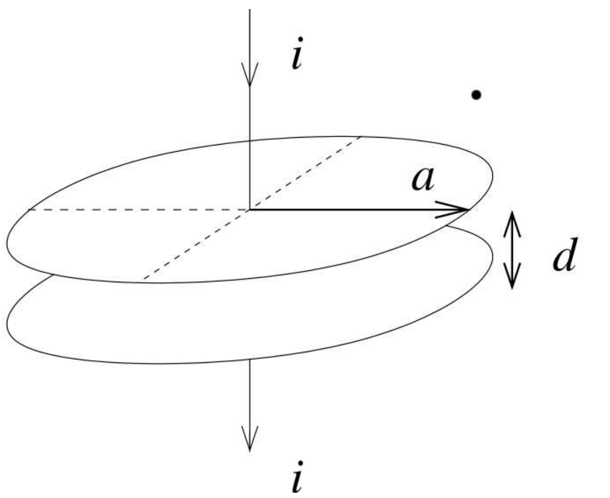
Dynamic Electromagnetism

Ex. 5.1: Conducting Cylinder II

Heald & Marion, ex 3.38.

Ex. 5.2: Capacitor

Consider a parallel-plate capacitor consisting of two large parallel perfectly conducting discs a distance d apart and separated by a vacuum. The discs have radius a. At time $t = 0$, the capacitor is connected to a battery, such that a constant current i starts to flow onto the upper plate. (The same current flows out of the lower plate.) When answering the questions below you may neglect edge effects. You may give all your answers to leading order in the small ratio a/ct.



a) What are the electric and magnetic fields in the space between the capacitor plates, as a function of time? Be sure to specify magnitude and direction.

b) Calculate the total field energy density for the space between the capacitor plates. Do the electric and magnetic fields contribute equally to the energy density?
Hint: use the inequality $t \gg a/c$.

c) Calculate the Poynting vector $\mathbf{S}$ for the space between the capacitor plates, and show what form the energy conservation law takes in this case.

d) Find the Maxwell Stress Tensor for the space between the capacitor plates.
Hint: Pay attention to the fact that $a \ll ct$. *Argue that the magnetic field does not contribute to the stress tensor in this limit.*

e) Use the Maxwell Stress Tensor to find the magnitude and direction of the force per unit area the plates exert on each other.

Ex. 5.3: Current Loop

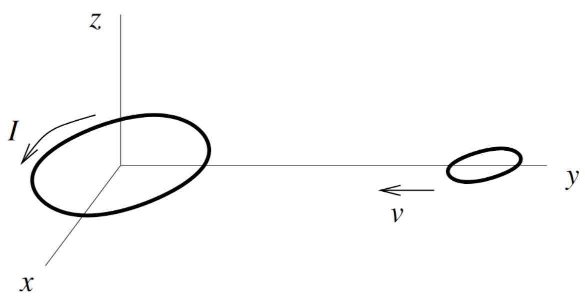
A circular loop with radius a carries a current I. We choose a coordinate system such that the current loop is situated in the xy plane, the center of the loop being at the origin of the coordinate system.

a) At a distance $r \gg a$ from the loop, the magnetic field $\mathbf{B}$ reads

$$\mathbf{B} = \frac{b}{r^3}\left(2\mathbf{e}_r \cos\theta + \mathbf{e}_\theta \sin\theta\right).$$

Derive this equation for the magnetic field and calculate the prefactor b.

b) A (second) conducting loop, with radius a', is held in the xy plane, with its origin on the y axis, to the right of the (first) current-carrying loop. This second loop has resistance R. It is moved towards the current-carrying loop at a constant velocity $\mathbf{v} = -v\mathbf{e}_2$ with $v \ll c$. Calculate the electromotive force ε in the moving loop. (You may assumethat the distance between the two loops is much larger than their radius.)



c) What is the power dissipated in the moving loop?

d) What is the force exerted on the moving loop?

Ex. 5.4: Rotating conducting sphere

Heald & Marion Ex 4-7

Ex. 5.5: Energy of uniformly charged sphere

Heald & Marion, ex 4-19

Ex. 5.6: Coulomb Gauge

Heald & Marion ex. 4-14

Suggested Heald & Marion Problems for further study:

4 - 1,3,14,15,17,23
5 - 3,4