incident $\begin{cases} \vec{E}_0 = E_0^\circ e^{i(k_I z - \omega t)} \hat{e}_x \\ \vec{H}_0 = n_I \hat{e}_z \times \vec{E}_0 = \underbrace{n_I E_0^\circ}_{H_0^\circ} e^{i(k_I z - \omega t)} \hat{e}_y \end{cases}$

reflected $\begin{cases} \vec{E}_1 = -E_1^\circ e^{i(-k_I z - \omega t)} \hat{e}_x \\ \vec{H}_1 = -n_I \hat{e}_z \times \vec{E}_1 = \underbrace{n_I E_1^\circ}_{H_1^\circ} e^{i(-k_I z - \omega t)} \hat{e}_y \end{cases}$

transmitted $\begin{cases} \vec{E}_2 = E_2^\circ e^{i(k_{II} z - \omega t)} \\ \vec{H}_2 = n_{II} \hat{e}_z \times \vec{E}_2 = \underbrace{n_{II} E_2^\circ}_{H_2^\circ} e^{i(k_{II} z - \omega t)} \hat{e}_y \end{cases}$

How do we relate $E_2^\circ, E_1^\circ, E_0^\circ$? Use Boundary Conditions

$E_{\parallel}$ is Continuous $(= E_x)$

$D_\perp$ is Continuous $(D_z \hat{e}_z = 0$ anyway$)$

$H_{\parallel}$ is Continuous $(= H_y)$

$B_\perp$ is Continuous (But $B_z \hat{e}_z = 0$ anyway)

$E_0^\circ - E_1^\circ = E_2^\circ$

$H_0^\circ + H_1^\circ = H_2^\circ \Rightarrow n_I E_0^\circ + n_I E_1^\circ = n_{II} E_2^\circ$

Solve for E_1° & E_2° in terms of E_0°

$E_1^\circ = \dfrac{n_{II} - n_I}{n_{II} + n_I} E_0^\circ \qquad H_1^\circ = n_I E_1^\circ$

$E_2^\circ = \dfrac{2 n_I}{n_{II} + n_I} E_0^\circ \qquad H_2^\circ = n_{II} E_2^\circ$

Lets look at some limits:

if $n_I = n_{II}$ $\quad$ $\left.\begin{array}{ll} E_1^\circ = 0 & H_1^\circ = 0 \\ E_2^\circ = E_0^\circ & H_2^\circ = H_0^\circ \end{array}\right\}$ no reflected wave; all is transmitted

if $n_{II} \gg n_I$ $\quad$ i.e. medium II is more optically dense than medium I almost everything is reflected. & reflected wave $\vec{E}_1$ gets a π phase shift. Reflected wave for $\vec{H}$ remains in phase.

if $n_{II} \ll n_I$ $\quad$ Then the transmitted wave has twice the amplitude of the incident wave.

reflected E field stays in phase

reflected H field picks up a phase of π since E_1° is negative

in both cases transmitted waves have the same phase as the incident wave.

Some Comments:

Ave Energy flux:

$$\langle \vec{S}_0 \rangle = \frac{c}{8\pi} \mathrm{Re}\, \vec{E}_0^\circ \times \vec{H}_0^{\circ *} = \frac{n_I c}{8\pi} |E_0^\circ|^2 \hat{e}_z \quad \text{incident}$$

$$\langle \vec{S}_1 \rangle = -\frac{c n_I}{8\pi} |E_1^\circ|^2 \hat{e}_z \quad \text{reflected}$$

$$\langle \vec{S}_2 \rangle = \frac{c}{8\pi} n_{II} |E_2^\circ|^2 \hat{e}_z \quad \text{transmitted}$$

Power reflection Coefficient

$$R = \frac{\langle \vec{S}_1 \rangle \cdot (-\hat{e}_z)}{\langle \vec{S}_0 \rangle \cdot \langle \hat{e}_z \rangle} = \frac{|E_1^\circ|^2 n_I}{|E_0^\circ|^2 n_I} = \left(\frac{n_{II} - n_I}{n_{II} + n_I} \right)^2$$

Power transmitted Coefficient

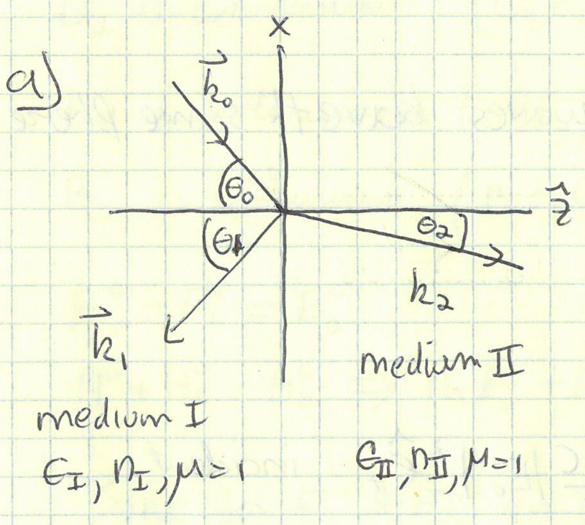
$$T = \frac{\langle \vec{S}_2 \rangle e_z}{\langle \vec{S}_0 \rangle e_z} = \frac{|E_2^\circ|^2 n_{II}}{|E_0^\circ|^2 n_I} = \frac{4 n_I n_{II}}{n_{II} + n_I}$$

Energy Conservation $\quad R + T = 1$

Ex: glass has $n_{II} = 1.5 \quad T = 0.96$ <u>at each surface</u>

Lets move on to more interesting case of oblique incidence

a)



assume $\vec{k}$ is in xz plane "plane of incidence"

$$\vec{E}_2 = \vec{E}_2^\circ e^{i(\vec{k}_2 \vec{r} - \omega t)}$$

$$\vec{H}_2 = \vec{H}_2^\circ e^{i(\vec{k}_2 \vec{r} - \omega t)} \qquad \vec{H}_2^\circ = n_{II} \hat{e}_{k_2} \times \vec{E}_2^\circ$$

The Equations

$$\vec{E}_0 = \vec{E}_0^\circ e^{i(\vec{k}_0 \vec{r} - \omega t)}$$

$$\vec{H}_0 = \vec{H}_0^\circ e^{i(\vec{k}_0 \vec{r} - \omega t)} \qquad \vec{H}_0^\circ = n_I \hat{e}_{k_0} \times \vec{E}_0^\circ$$

$$\vec{E}_1 = \vec{E}_1^\circ e^{i(\vec{k}_1 \vec{r} - \omega t)}$$

$$\vec{H}_1 = \vec{H}_1^\circ e^{i(\vec{k}_1 \vec{r} - \omega t)} \qquad H_1^\circ = n_I \hat{e}_{k_1} \times E_1^\circ$$

Boundary Conditions

Tangential Components of $\vec{E}, \vec{H}$ are Continuous at $z = 0$ for all x, y

Since $\left.\begin{matrix}\vec{E}_0\\ \vec{H}_0\end{matrix}\right\} \propto e^{i\vec{k}_0\cdot\vec{x}}$, $\left.\begin{matrix}\vec{E}_1\\ \vec{H}_1\end{matrix}\right\} \propto e^{i\vec{k}_1\cdot\vec{x}}$, $\left.\begin{matrix}\vec{E}_2\\ \vec{H}_2\end{matrix}\right\} \propto e^{i\vec{k}_2\cdot\vec{x}}$

at $z=0$ we must have $k_{0x} = k_{1x} = k_{2x}$ (Continuity)

~~Since~~ also $(k_{0y} = k_{1y} = k_{2y} = 0)$

also: $|\vec{k}_0| = \frac{\omega}{c} n_I \quad |\vec{k}_1| = \frac{\omega}{c} n_I \quad |\vec{k}_2| = \frac{\omega}{c} n_{II}$

$$k_{0x} = -k_0 \sin\theta_0 = k_{1x} = -k_1 \sin\theta_1$$

$\theta_0 = \theta_1$ Specular reflection

$$k_{2x} = -|\vec{k}_2| \sin\theta_2$$

$$\frac{\sin\theta_0}{\sin\theta_2} = \frac{k_{0x}/k_0}{k_{2x}/k_2} = \frac{n_{II}}{n_I}$$ Snells Law

How do we get Coeff $\vec{E}_1, \vec{E}_2, \vec{H}_1, \vec{H}_2$

B.C. at interface $\vec{D}_\perp, \vec{B}_\perp$ Continuous

$\vec{E}_{\parallel}, \vec{H}_{\parallel}$ Continuous

Second Condition + Snells law implies first Condition

$\Rightarrow$ need to solve

$$(\vec{E}_0 + \vec{E}_1)\cdot\hat{e}_x = \vec{E}_2\cdot\hat{e}_x$$

$$(\vec{E}_0 + \vec{E}_1)\cdot\hat{e}_y = \vec{E}_2\,\hat{e}_y$$

$$(\vec{H}_0 + \vec{H}_1)\,\hat{e}_x = \vec{H}_2\,\hat{e}_x$$

$$(\vec{H}_0 + \vec{H}_1)\,\hat{e}_y = \vec{H}_2\,\hat{e}_y$$

Four equations four unknowns

Unique Solution

Solution takes simple form if

① $\vec{E}_0^\circ$ in xz plane (in plane of incidence)

② $\vec{E}_0^\circ$ in y direction ($\perp$ to plane of incidence)

General Case is superposition of these two cases.

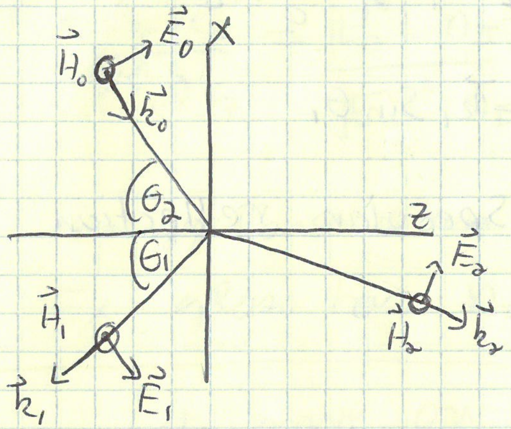
Case 1) All $\vec{E}_j$ in xz plane

all $\vec{H}_j$ along y

only two of the four equations remain

$$E^\circ_{0x} + E^\circ_{1x} = E^\circ_{2x}$$

$$\hat{e}_y \cdot \left(n_I\, \hat{e}_{\vec{k}_0} \times \vec{E}^\circ_0 + n_I\, \hat{e}_{\vec{k}_1} \times \vec{E}^\circ_1\right) = \hat{e}_y\, n_{II} \left(\hat{e}_{\vec{k}_2} \times \vec{E}^\circ_2\right)$$



$$E^\circ_0 \cos\theta_0 - E^\circ_1 \cos\theta_1 = E^\circ_2 \cos\theta_2$$

& $n_I\left(E^\circ_0 + E^\circ_1\right) = n_{II} E^\circ_2$

Fresnel equations:

$$E^\circ_1 = \frac{\cos\theta_0 - \frac{n_I}{n_{II}} \cos\theta_2}{\cos\theta_0 + \frac{n_I}{n_{II}} \cos\theta_2} E^\circ_0 = \frac{\tan(\theta_0 - \theta_2)}{\tan(\theta_0 + \theta_2)}$$

$$E^\circ_2 = \frac{\frac{n_I}{n_{II}}\, 2\cos\theta_0}{\cos\theta_0 + \frac{n_I}{n_{II}} \cos\theta_2} E^\circ_0$$

$$= \frac{2\cos\theta_0 \sin\theta_2}{\sin(\theta_0 + \theta_2)\cos(\theta_0 - \theta_2)} E^\circ_0$$

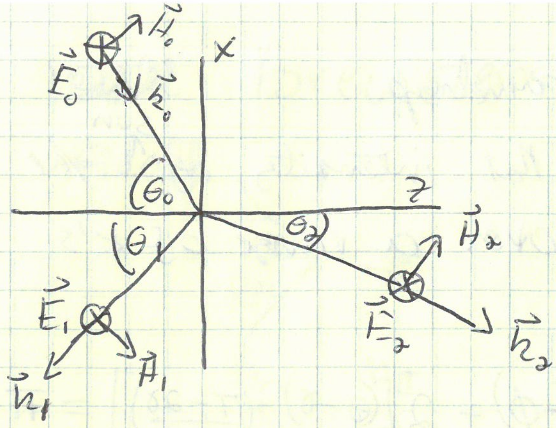
Case 2) All $\vec{E}_j$ along y

all H_j in xz plane

again only two equations remain

$$E^\circ_{0y} + E^\circ_{1y} = E^\circ_{2y}$$

$$\hat{e}_x \cdot \left(n_I\, \hat{e}_{\vec{k}_0} \times \vec{E}_0 + n_I\, \hat{e}_{\vec{k}_1} \times \vec{E}_1\right) = \hat{e}_x \cdot \left(n_{II}\, \hat{e}_{\vec{k}_2} \times \vec{E}_2\right)$$



$E_0^\circ + E_1^\circ = E_2^\circ$

$n_I(E_0^\circ \cos\theta_0 - E_1^\circ \cos\theta_1) = n_{II} E_2 \cos\theta_2$

Fresnel equations

$$E_1^\circ = \frac{\cos\theta_0 - \frac{n_{II}}{n_I}\cos\theta_2}{\cos\theta_0 + \frac{n_{II}}{n_I}\cos\theta_2} E_0^\circ = \frac{\sin(\theta_2 - \theta_0)}{\sin(\theta_2 + \theta_0)} E_0^\circ$$

$$E_2^\circ = \frac{2\cos\theta_0}{\cos\theta_0 + \frac{n_{II}}{n_I}\cos\theta_2} E_0^\circ$$

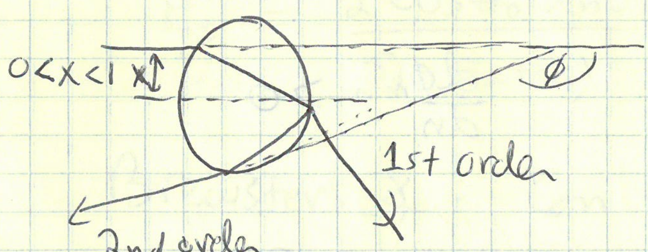
$$= \frac{2\cos\theta_0 \sin\theta_2}{\sin(\theta_2 + \theta_0)} E_0^\circ$$

talk about ~~red~~ tooth paste, milk, BWS

$\boxed{\text{Rainbows}}$

Show pics of rainbow at grassroots festival
+ Rainbow mythology

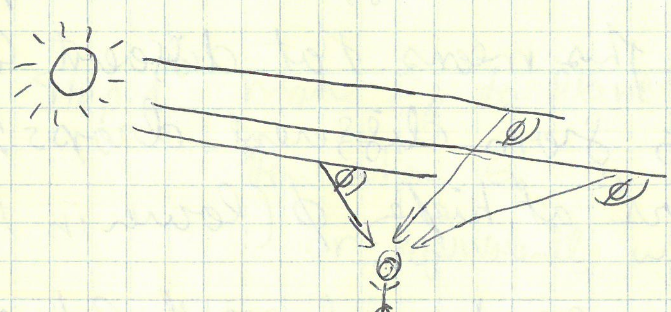
1) Sun light is reflected when passing through raindrops



Total deflection ∢ ϕ depends on order m & X. for water $n = 1.333$

$\frac{dn}{dw} > 0$ Color separation

②



what is the intensity of sunlight as a function of ϕ?

$\phi = 0$: huge intensity from light that has not passed through any drops

Q: are there any intensity peaks at other ϕ?

③ 1st order pass through raindrop

Long calculation – Result is that intensity of sun light is smoothly distributed over a range of ∡'s

④ Second order passage

$$(\pi-\phi) + 2\left[(\theta-\gamma) + \left(\frac{\pi-2\gamma}{2}\right)\right] = \pi$$

$$\phi = 2(\theta-\gamma) + (\pi-2\gamma)$$

$$= \pi + 2\theta - 4\gamma$$

from Snell's law

$$= \pi + 2\theta - 4\arcsin\left(\frac{\sin\theta}{n}\right)$$

$$x = \sin\theta$$

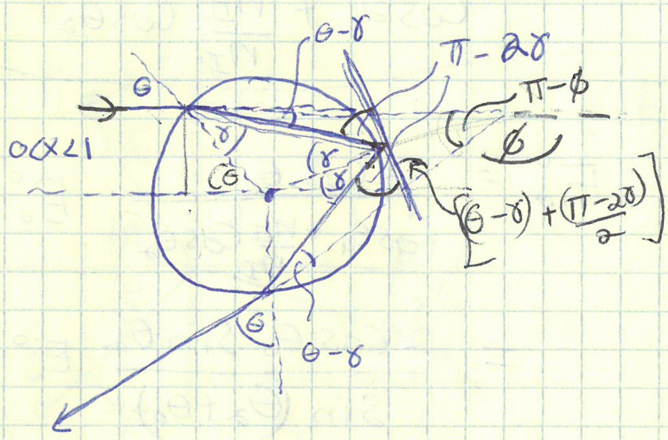
$$= \pi + 2\arcsin x - 4\arcsin\frac{x}{n}$$

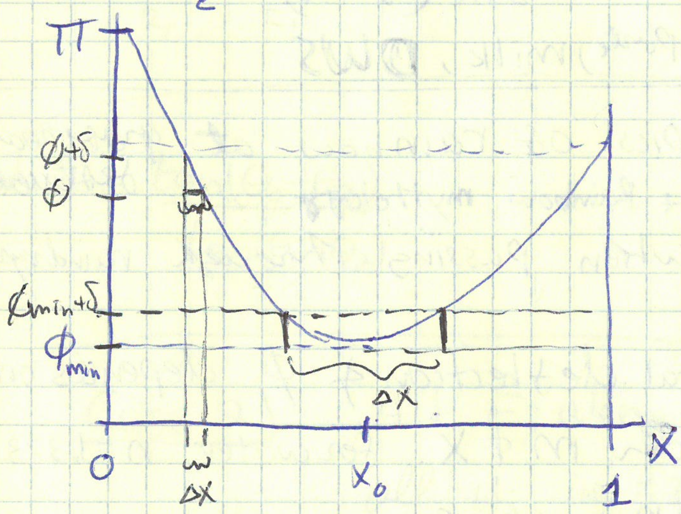
$\arcsin 0 = 0 \qquad \arcsin 1 = \frac{\pi}{2}$

ϕ_{min} at $x = \sqrt{\frac{4-n^2}{3}}$

$n = 1.333 \qquad \phi_{min} \simeq 138°$

$$\frac{d\phi_{min}}{dn} > 0$$



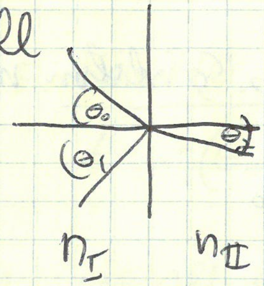


Since n depends on ω different colors appear at different ∡'s. This means that different colors must be coming from different drops! Higher freq occurs at higher φ (lower in the sky)

Why is a double rainbow have the colors switched?

~~recall~~ Brewsters $\measuredangle$

recall



Snells law

$$\frac{\sin\theta_0}{\sin\theta_2} = \frac{n_{II}}{n_I}$$

Specolor reflection

$\theta_0 = \theta_1$

Fresnell Eqs:

$$E_1^0 = \frac{\tan(\theta_0 - \theta_2)}{\tan(\theta_0 + \theta_2)} E_0^0$$

$$E_2^0 = \frac{2\cos\theta_0 \sin\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_0+\theta_2)} E_0^0$$

$\vec{E}_{\parallel}$ plane of incidence

$$E_1^0 = \frac{\sin(\theta_2 - \theta_0)}{\sin(\theta_2 + \theta_0)} E_0^0$$

$$E_2^0 = \frac{2\cos\theta_0 \sin\theta_2}{\sin(\theta_2+\theta_0)} E_0^0$$

$\vec{E}_{\perp}$ to plane of incidence

Brewster's $\measuredangle$: Can use polarizers to eliminate reflected Light at Brewster's $\measuredangle$

Step 1 make $E_{\parallel}$ to plane of incidence

Then when $\theta_0 = \theta_2$ or $\theta_0 + \theta_2 = \frac{\pi}{2}$

or equivalently when $\theta_1 + \theta_2 = \frac{\pi}{2}$

$$E_1^0 = 0$$

$\theta_0 = \theta_2$ trivial $n_I = n_{II}$

$\theta_0 + \theta_2 = \frac{\pi}{2}$: when reflected & refracted rays are $\perp$, no energy is carried by reflected ray

Since $n_{II} \sin\theta_2 = n_I \sin\theta_0$ this condition means

$$n_{II} \underbrace{\sin\left(\frac{\pi}{2} - \theta_0\right)}_{\cos\theta_0} = n_I \sin\theta_0$$

$$\tan\theta_B = \frac{n_{II}}{n_I} \qquad \theta_B \equiv \text{Brewster} \measuredangle$$

For $\vec{E}_0 \perp$ to plane of incidence $E_i^0 = 0$ only if $n_I = n_{II}$

Consequently, light reflected off an interface at θ_B is always linearly polarized $\perp$ to plane of incidence

glass $n_{II} = 1.5$ $\quad \theta_B \cong 56^\circ$

water $n_{II} = 1.33$ $\quad \theta_B \approx 53^\circ$

Aside show Demo Brewster $\measuredangle$ again talk about Brewster $\measuredangle$ microscopy

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Power reflection & transmission coefficients

$\vec{E}_0 \perp$ plane of incidence

$$R_\perp = \frac{\langle \vec{S}_1 \rangle \cdot (-\hat{e}_z)}{\langle \vec{S}_0 \rangle \cdot (\hat{e}_z)} = \frac{|E_1^0|^2}{|E_0^0|^2} = \frac{\sin^2(\theta_2 - \theta_0)}{\sin^2(\theta_2 + \theta_0)}$$

$$T_\perp = 1 - R_\perp = \frac{\langle \vec{S}_2 \rangle \cdot (\hat{e}_z)}{\langle \vec{S}_0 \rangle \cdot (\hat{e}_z)} = \frac{|E_1^0|^2}{|E_0^0|^2} = \cancel{\frac{\sin^2(\theta_2 - \theta_0)}{\sin^2(\theta_2 + \theta_0)}} \; \frac{n_{II} \cos\theta_2}{n_I \cos\theta_0}$$