PHYS 327 PRELIM 1

Prof. Itai Cohen, Fall 2009

Friday, 10/16/09 **Name:**

Read all of the following information before starting the exam:

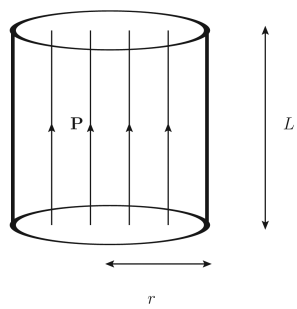
- Put your name on the exam **now**.
- Show all work, clearly and in order, if you want to get full credit.
- Circle or otherwise indicate your final answers.
- The first two problems are conceptual, the next two are computational.
- Question 3 (c) is a bonus question worth 10 points. The total exam score cannot exceed 100, but the bonus question can help you make up points lost elsewhere.

Problem #	Score
1	/25
2	/15
3	/20
3(c) Bonus	/10
4	/40
Total	/100

- It is your responsibility to make sure that you have all of the pages!
- Good luck!

Question 1: Tricky Electret [25 points]

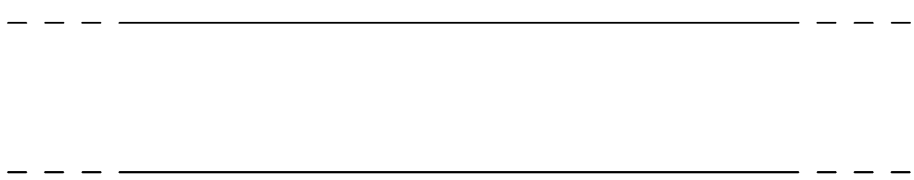
An *electret* is a material with a permanent dielectric polarization (the electric analog of a permanent magnet). Consider a cylindrical electret of radius r and length L, in which the polarization $\mathbf{P}$ is constant and parallel to the axis of the cylinder:



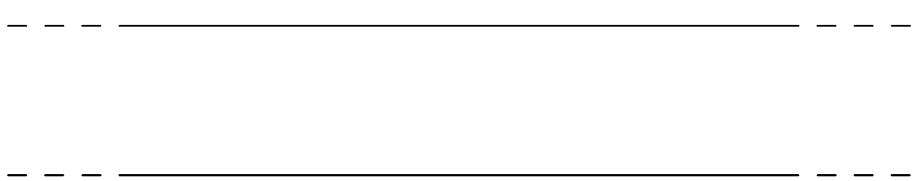
Draw careful sketches of the $\mathbf{E}$ and $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ fields inside and outside the electret for the following cases (next three pages):

$L \ll r$, i.e. treat r as being infinite..

(a) [5 points] Sketch the E-field

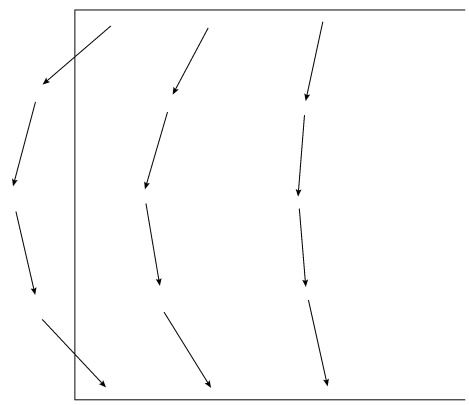


(b) [10 points] Sketch D-field and explain your answer in words. You must explain your answer to get credit.

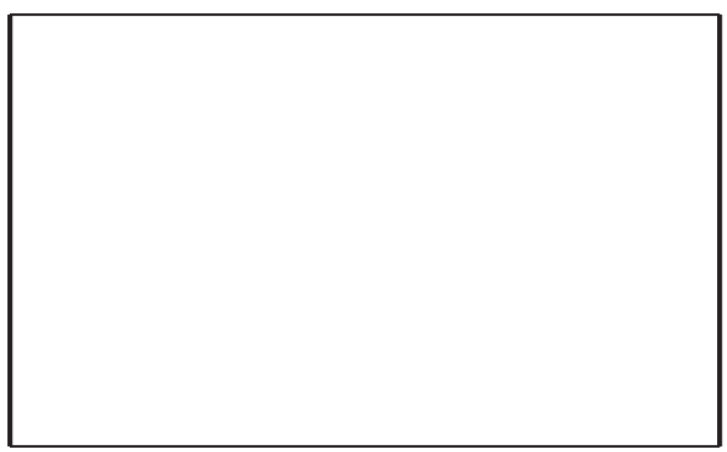


$L \sim r$

The E-field near the edge of the electret is shown in the sketch below:



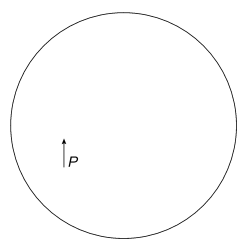
c) [10 points] Using vector addition, determine the D-field near the edge of the electret. Keep in mind that D-field lines are continuous if there is no free surface charge. Sketch the D-field lines below:



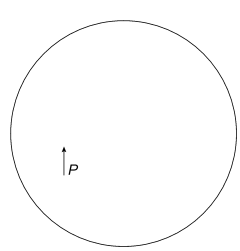
Question 2: Uniformly Polarized Sphere [15 points]

We know that the electric field of a uniformly polarized dielectric sphere is that of a perfect dipole (outside of the sphere).

(a) [5 points] Sketch the electric field inside and outside



(b) [10 points] Draw the equipotentials



Question 3: Spaced Out Bubbles [20 + 10 points]

While in the airlock (in a vacuum), Major Tom on the International Space Station decides to play bubbles by attaching his air hose to a soap bubble maker. As the bubbles float around Major Tom notices that they increase in size. He suspects that the bubbles are picking up some stray ions floating around in the airlock. You are a NASA engineer from Houston and Major Tom is telling you to help him solve the mystery of the growing bubbles. You better not mess with Major Tom.

Consider a bubble of radius R and surface tension γ, with total charge q distributed evenly across its surface.

Obviously this problem requires a balance of pressures. The pressure in the bubble is given by the ideal gas law $P_{\text{air}} = nkT/V$, where n is the number of "air particles" and V is the bubble's volume. The surface tension causes a pressure of magnitude $P_\gamma = 2\gamma/R$ which wants to collapse the bubble.

(a) [10 points] If ions are collecting on the surface of the soap bubble, find the corresponding electrostatic pressure P_{charge} as a function of q and R.

(b) [10 points] If all three pressures balance, the bubble is stable. Using this balance, find the charge q required for a given bubble radius.

(c) Bonus Question [10 points]: For some range of radii, the charge is ill-defined. Interpret this result.

Question 4: Quadrupole in Cavity [40 points]

Recall from lectures, for an *axial* quadrupole with rotational symmetry around the z axis, the quadrupole moment and potential are given by:

$$Q_{ij} = \begin{pmatrix} -\frac{1}{2}Q & & \\ & -\frac{1}{2}Q & \\ & & Q \end{pmatrix} \qquad \Phi^{(4)}(r,\theta) = \frac{1}{2}Q\frac{\frac{1}{2}(3\cos^2\theta - 1)}{r^3}$$

Consider a spherical cavity of radius a in a medium of dielectric constant ϵ. At the center, we have an axial quadrupole. Assume the quadrupole is infinitesimal, so the potential as $r \to 0$ is just given by $\Phi^{(4)}$. Also assume the potential vanishes as $r \to \infty$.

Solve the Laplace Equation to find the potential inside Φ^{int} *and* outside Φ^{ext} the cavity, by following these steps [8 points each]:

(a) Write down the Boundary Conditions for $\Phi^{\text{ext}}, \Phi^{\text{int}}$ for $r \to 0$, $r = a$ and $r \to \infty$.
Hint: you may assume the dielectric is linear.

The general solution to Laplace's Equation with cylindrical symmetry is

$$\Phi = \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + B_\ell \frac{1}{r^{\ell+1}} \right) P_\ell(\cos\theta)$$

where the Legendre Polynomials are given by

$$P_o(x) = 1, \qquad P_1(x) = x, \qquad P_2(x) = \frac{1}{2}(3x^2 - 1), \qquad P_3(x) = \frac{1}{2}(5x^3 - 3x), \quad \ldots$$

(b) Apply the $r \to 0$ BC to Φ^{int} to determine which terms in the expansion survive.

(c) Similarly, apply the $r \to \infty$ BC to Φ^{ext}

(d) Now apply the $r = a$ BCs to solve for Φ^{int} and Φ^{ext}.

(e) What happens to your solution as $\epsilon \to 1$? Does this make sense?