Cornell University, Physics Department — Fall 2014
PHYS-3341 Statistical Physics — Prof. Itai Cohen

Problem Set 1

Due Friday Sept. 12, 2014

1.1 Birthday Problem

Suppose there are N people in a room. What is the probability that at least two of them share the same birthday - the same day of the same month?

1.2 Russian Roulette

Reif §1.5

1.3 1-D Random Walk

Reif §1.6

1.4 Alternative Analysis of the 1-D Random Walk

In lecture and in the text, we evaluated the probability distribution for taking n_+ of N total steps in the $+x$ direction $P_N(n_+)$ and, by substituting $m = 2n_+ - N$, the distribution of the net number of steps m in $+x$ direction. Using this distribution, we calculated the mean number of steps and the standard deviation.

Another approach is to consider the probability distribution of the individual steps, as follows. Assume that all steps have the same length l, and that the probability of taking steps in the $+x$ and x directions are p and q, respectively.

(a) Sketch the probability distribution of a single step s_i versus x. Does this correspond to any of the standard probability distributions we have considered so far?

(b) What are the mean and standard deviation of the distribution of s_i?

(c) The total displacement $x_N = ml$ after N steps can be expressed as a sum of N statistically independent random variables s_i. Evaluate the mean number of steps taken in the $+x$ direction. (Hint: What is the mean of a sum of independent random variables?)

(d) Evaluate the standard deviation of m. (Hint: What is the mean of a product of statistically independent random variables?)

(e) Similarly, evaluate the expectation value of m^3 and m^4. Compare your answers with the previous question.

(f) Arguing based upon the Central Limit Theorem, what would you expect the probability distribution of m to look like in the limit of large N, i.e., when you add up a very large number of statistically independent random variables each with the distribution sketched in (a)? What should be the mean and standard deviation of this distribution?

1.5 Telephone Problem

Reif §1.15

1.6 3-D Isotropic Scattering

Reif §1.18

1.7 Uniform Distributions on Circles and Spheres

Reif §1.24

1.8 Waiting Times

On a certain one-way road, the average numbers of passing cars and buses are equal: each hour, on average, there are 12 buses and 12 cars passing by. The buses are scheduled: each bus appears exactly 5.0 minutes after the previous one. On the other hand, the cars appear at random. In a short interval dt, the probability that a car comes by is dt/τ, with $\tau = 5.0$ minutes.

An observer is counting the cars and the buses.

(a) Verify that each hour the average number of cars that passes the observer is 12.

(b) What is the probability $P_{bus}(n)$ that n buses pass the observer in a randomly chosen 10 minute time interval? And what is the probability $P_{car}(n)$ that n cars pass the observer in the same time interval?

(c) What are the mean and variance of the probabilities calculated under (b)?

(d) What is the probability distribution $P_{bus}(\Delta t)$ and $P_{car}(\Delta t)$ for the time interval Δt between two successive buses and cars, respectively? What are the mean and variance of these distributions?

(e) If another observer arrives at the road at a randomly chose time, what is the probability distribution of the time Δt she has to wait for the first bus to arrive? And what is the probability distribution of the time she has to wait for the first car to pass by? What are the mean and variance of these distributions? How do your answers compare to (d)?

The same ideas can be applied to the motion of electrons in a metal with many impurities, or for the motion of gas molecules in a dilute gas. In the absence of electric or magnetic fields and according to classical mechanics, electrons in a metal move in straight lines. At certain times, the electrons collide with the impurities and randomly change their direction of motion. The impurities are static (they don't move) and collisions between the electrons and the impurites can be considered elastic.

We can now ask ourselves what the distribution of those collision times will be. Above we have considered two paradigms for such a distribution: collisions at equally spaced times (like buses passing by) or collisions at random times (like cars passing by). According to the first scenario, the time between collisions is τ and does not fluctuate. According to the second scenario, each electron has a probability dt/τ to collide in a short time interval dt.

(f) Which scenario is more relevant for electrons in a dirty metal or for gas molecules in a dilute gas?

(g) Taking the scenario of randomly distributed collision times, what is the probability distribution of the time interval Δt between collisions? What is the mean time between collisions?

(h) At any time t during the electron's motion what is the probability distribution of the time Δt_{next} till the next collision? And of the time Δt_{prev} from the previous collision? What are the mean values of Δt_{next} and Δt_{prev}? (You may assume that collisions are instantaneous, so that you can ignore the possibility that an electron is "in a collision" at the time t.)

(i) Are your answers to (h) consistent with your answers to (g)? Explain!

1.9 Questions

Email three questions about the reading assignments for the coming week to ic64@cornell.edu and ydy3@cornell.edu.