Problem Set 3

Due Monday Sept 29, 2014

3.1 Boundary Conditions

The energy eigenstates of a point particle of mass m in a cubic containere of size L are labeled by three positive integers n_x, n_y, and n_z,

$$\psi_{n_x,n_y,n_z}(x, y, z) = \left(\frac{2}{L}\right)^{3/2} \sin\frac{\pi n_x x}{L} \sin\frac{\pi n_y y}{L} \sin\frac{\pi n_z z}{L} \tag{1}$$

and the corresponding energy is

$$E_{n_x,n_y,n_z} = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2).$$

The wavefunction (1), which has the form of a *standing wave*, is calculated using the boundary condition that $\psi(x, y, z) = 0$ for x, y, z on the boundary of the walls of the container. This boundary condition is referred to as the "Dirichlet boundary condition".

Sometimes it is easier to use a different boundary condition. One often uses the so-called "periodic boundary condition". Instead of requiring the wavefunction vanishes on the walls of the contiainer, one requires *periodicity*,

$$\psi(x + L, y, z) = \psi(x, y + L, z) = \psi(x, y, z + L) = \psi(x, y, z).$$

For this boundary condition, wavefunctions are *travelling waves*. They are labeled by integers n_x, n_y, n_z, which can be positive, negative and zero.

$$\psi_{n_x,n_y,n_z}(x, y, z) = L^{-3/2} e^{2\pi i(n_x x+n_y y+n_z z)/L}.$$

The corresponding energy eigenvalue is

$$E_{n_x n_y n_z} = \frac{2\hbar^2\pi^2}{mL^2}(n_x^2 + n_y^2 + n_z^2).$$

(a) Calculate the microcanonical paritition function $\Omega(E)$ for an ideal gas of $N \gg 1$ non-interacting distinguishable point particles in a cubic container of size L with periodic boundary conditions. How does your answer compare to the case of Dirichlet boundary conditions?

(b) Another boundary condition is the "Neumann boundary condition", in which one requires that the normal derivative of wavefunction vanishes on the walls of the container. What would be the microcanonical paritition function in this case? You don't need to show a full calculation.

3.2 Magnetization with Temperature

Reif §3.2

3.3 Spin Systems in a Magnetic Field

Reif §3.3

3.4 Mixture of Ideal Gases

Reif §3.5

3.5 Email 3 Questions