Problem Set 6

Due Friday Oct 24, 2014

6.1 Harmonic Oscillator

Reif §6.1

6.2 Harmonic Oscillator

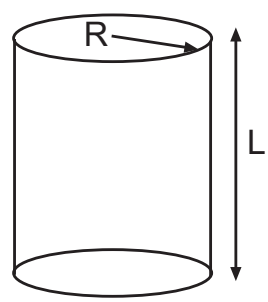
Reif §6.6

6.3 Centrifuge

An ideal gas is enclosed in a centrifuge with radius R and height L. The centrifuge rotates at angular frequency ω. Choosing the z axis along the axis of the centrifuge, one may describe the dynamics of a gas molecule by the (classical) Hamiltonian function

$$H = \frac{p^2}{2m} - \omega(xp_y - yp_x).$$

Here m is the mass of a gas molecule.



(a) Show that the partition function Z_1 for one gas molecule is equal to

$$Z_1 = \left(\frac{2\pi m}{h^2\beta}\right)^{3/2} \frac{2\pi L}{m\beta\omega^2}\left(e^{m\beta\omega^2 R^2/2} - 1\right).$$

Hint: First integrate over momentum, then integrate over the spatial coordinates by shifting to cylindrical coordinates.

(b) Calculate the partition function Z if there are N identical gas molecules in the cylinder. You may keep the single-molecule partition function Z_1 in your answer.

(c) Calculate the Helmholtz free energy F of the gas in the centrifuge. You may keep the single-molecule partition function function Z_1 in your answer.

(d) Show that the total force on the outer wall of the centrifuge can be calculated as the derivative of the Helmholtz free energy F and calculate that force.

(e) Calculate the density of gas molecules $n(r)$ as a function of the distance r to the axis of the centrifuge.

6.4 Statistical Entropy

In class we introduced the concept of Statistical or Informational Entropy, but we did not show that this was the *same* quanitity as the thermodynamic entropy.

For the canonical ensemble, the entropy can be calculated as

$$S = \frac{\partial}{\partial T} k \ln Z.$$

In section §6.6 Reif shows that this expresssion for the entropy can be rewritten as

$$S = -k \sum_r P_r \ln P_r,$$

where the summation is over all microstates r and $P_r = Z^{-1} exp(-E_r/kT)$ is the probability to find the system in the microstate r, the same as the statistical entropy we defined in section.

(a) Show that the same expression also holds for the entropy in the microcanonical ensemble

(b) Now answer exercises §6.13 and §6.15 from Reif.

6.5 Ideal Gas in a Gravitational Field

Reif §7.2

6.6 Send three questions