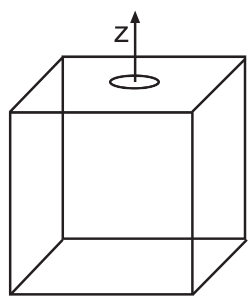
Problem Set 7

Due Halloween, Oct 31, 2014

7.1 Dilute Gas in an Enclosure

A container of volume V contains N molecules of dilute gas at a temperature T. Some molecules can escape into a vacuum by effusing through a small hole in one of the walls of the container (see figure). The hole has an area A, and its diameter is much smaller than the mean free path for collisions between the gas molecules. Choose the z direction so as to point along the outward normal to the plane of this hole. The mass of a molecule is denoted m. You can use classical mechanics to describe the motion of the molecules. The density of



gas molecules at position $\vec{r}$ and velocity $\vec{v}$ is given by the Maxwell Boltzmann distribution,

$$n(\vec{r}, \vec{v}) = \frac{N}{V}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}.$$

(a) What is the mean velocity component $\vec{v}_z$ of a molecule? What is the mean value of $|v_z|$? What is the mean value of v_z^2?

(b) Denote the number of molecules escaping into the vacuum per unit time by Φ. Find an expression for Φ in terms of A, N, V, T, and/or the pressure p of the gas in the container. *Hint: you can use your answer to (a).*

(c) Derive a relation between the pressure p of the gas, the mean velocity component $\overline{v_{z,esc}}$ of a molecule which has escaped into the vacuum, and the flux Φ of escaping molecules.

(d) Now compare the mean velocity component $\overline{v_{z,esc}}$ of a molecule which has escaped into the vacuum to the mean absolute velocity component $\overline{|v_z|}$ for molecules in the container. Which of the following relations is true? Why?

1. $\overline{v_{z,esc}} > \overline{|v_z|}$,

2. $\overline{v_{z,esc}} = \overline{|v_z|}$,

3. $\overline{v_{z,esc}} < \overline{|v_z|}$.

7.2 Two-Dimensional Gas

Reif §7.7

7.3 More Oscillators

Reif §7.10

7.4 Graphite

Reif §7.11

7.5 Ferro Fluids

Reif §7.14

7.6 Mean Values

Reif §7.19

7.7 Isotope Separation

Reif §7.26

7.8 Send three questions