PHYS 3341 Quiz 1

Prof Itai Cohen, Fall 2013
Thursday Oct. 3rd, 2013 **Name:**

Read all of the following information before starting the exam:

- Put your name on the exam **now**.
- Show all work, clearly and in order, if you want to get full credit.
- Circle or otherwise indicate your final answers.

Problem #	Score
1	/10
2	/40
Total	/50

- It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Ergodicity (*10 points*) Explain in your own words the utility of ergodicity is in the development of statistical mechanics.

The assumption of ergodicity - that over long periods the system spends equal time in all accessible configurations - allows us to do away with the particulars of how the system traverses its phase space and what its initial conditions are. It sets up the basic tools of calculation - ensembles, partition functions, and so on - that allow us to calculate macroscopic parameters without dealing with microscopic states.

2. Streched Hot Rod (*40 points*) An elastic rod of length L at a temperature T has a small amount of work done **ON** the system $-dW$ given by

$$-dW = (-\alpha(T - T_0) + k_T(L - L_0))dL$$

where T_0 and L_0 are the reference temperature and length, k_T is the spring constant of the rod at constant T and α and k_T are positive constants.

a. (*5 pts*) What is the generalized force F that is conjugate to the external parameter L? (hint: what's the relation between dE and dW? are the other terms related to L?)

$$\boxed{F = -\left(\frac{\partial E}{\partial L}\right) = \frac{\partial W}{\partial L} = \alpha(T - T_0) - k_T(L - L_0)}$$

b. (*5 pts*) What is the thermodynamic relation for the energy? What Maxwell relation follows?

$$\boxed{dE = TdS - FdL}$$

This gives us the Maxwell relation

$$\boxed{\left(\frac{\partial T}{\partial L}\right)_S = -\left(\frac{\partial F}{\partial S}\right)_L}$$

c. (*5 pts*) Construct a thermodynamic potential for which the fundamental variables are T and L. What Maxwell relation follows?

Starting with $dE = TdS - FdL$ we can perform the Legendre transform

$$\boxed{\mathcal{F} = E - TS \Rightarrow d\mathcal{F} = -SdT - FdL}$$

This gives us the Maxwell relation

$$\boxed{\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial F}{\partial T}\right)_L}$$

d. (*15 pts*) Assume the rod initially has length L_1 and temperature T_1. In an adiabatic process, it is stretched to length L_2. Using one of the Maxwell relations above, find T_2, the temperature of the rod at the end of the process.
Express T_2 in terms of $\alpha, kT, L_0, T_0, L_1, T_1$ and C_L, the rod's heat capacity at constant length. (hint: make use of C_L. Remember $\frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$)

We have

$$\left(\frac{\partial T}{\partial L}\right)_S = -\left(\frac{\partial F}{\partial S}\right)_L = -\alpha\left(\frac{\partial T}{\partial S}\right)_L = -\alpha\left(\frac{C_L}{T}\right)^{-1}$$

and so

$$\begin{aligned}
\int_{T_1}^{T_2} \frac{1}{T}\mathrm{dT} &= -\frac{\alpha}{C_L}\int_{L_1}^{L_2} \mathrm{dL} \\
\ln(T_2/T_1) &= -\frac{\alpha}{C_L}(L_2 - L_1)
\end{aligned}$$

$$\boxed{T_2 = T_1 e^{-\frac{\alpha(L_2 - L_1)}{C_L}}}$$

e. (*10 pts*) Next, the temperature is held fixed at T_2 and the rod is compressed back to L_1. Find the change in entropy in this process.

We have

$$\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial F}{\partial T}\right)_L = \alpha$$

$$\boxed{\Delta S = -\alpha(L_2 - L_1)}$$

Possible Useful Formulas

$$n! \approx n^n e^{-n}\sqrt{2\pi n} \qquad \Gamma(n+1) \approx n^n e^{-n}\sqrt{2\pi n}$$

$$\binom{N}{n} p^n q^{N-n} \approx \frac{1}{\sqrt{2\pi Npq}} e^{-(n-Np)^2/(2Npq)}$$

$$\int_0^\infty dx e^{-ax^2} = \frac{\sqrt{\pi}}{2} a^{-1/2}$$

$$\int_0^\infty dx x e^{-ax^2} = \frac{1}{2} a^{-1}$$

$$\int_0^\infty dx x^2 e^{-ax^2} = \frac{\sqrt{\pi}}{4} a^{-3/2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\frac{d}{dx}\cosh(x) = \sinh(x) \qquad \frac{d}{dx}\cosh(x) = \sinh(x)$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \frac{d}{dx}\tanh(x) = \frac{1}{\cosh^2(x)}$$

$$PV = NkT \qquad E = \frac{d}{2}NkT$$

$$dE = TdS - pdV \qquad dH = TdS + Vdp$$

$$dF = -SdT - pdV \qquad dG = -SdT + Vdp$$

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V \qquad C_p = T\left(\frac{\partial S}{\partial T}\right)_P$$

$$\beta = \frac{\partial \ln \Omega}{\partial E} \qquad S = k \ln \Omega$$

$$\bar{X}_a = \frac{1}{\beta}\frac{\partial \ln \Omega}{\partial x_a} \qquad \bar{p} = \frac{1}{\beta}\frac{\partial \ln \Omega}{\partial V}$$

$$\bar{E} = -\frac{\partial \ln Z}{\partial \beta} \qquad S = k \ln Z - k\beta\left(\frac{\partial \ln Z}{\partial \beta}\right)_{V,N}$$

$$p = kT\left(\frac{\partial \ln Z}{\partial V}\right)_{T,N} \qquad \mu = -kT\left(\frac{\partial \ln Z}{\partial N}\right)_{T,V}$$

$$\bar{N} = kT\left(\frac{\partial \ln \Xi}{\partial \mu}\right)_{V,T} \qquad S = k \ln \Xi - k\beta\left(\frac{\partial \ln \Xi}{\partial \beta}\right)_{V,\mu}$$

$$pV = kT \ln \Xi$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_o} \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0 \qquad \nabla \times H = J + \epsilon \frac{\partial E}{\partial t}$$

$$108 = 4 + 8 + 15 + 16 + 23 + 42$$

$$v_{\text{swallow}}(\text{unladen}) = \text{African or European?}$$

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