Quiz 1

Question 1: Triangular Prism Cavity in a Polarized Medium (20 pts)

Suppose an infinite *linear* dielectric medium contains a cavity in the shape of a right isoceles triangular prism. The dielectric has uniform polarization $\boldsymbol{P}$ and hence uniform electric field $\boldsymbol{E}$, as shown below in cross-sectional view of the cavity. The dielectric constant is $\varepsilon = 2$.

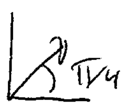
a) (5pts) Find the bound surface charge density on each face in terms of P.

$\xrightarrow{P}$ − +

+1 $\sigma_b = \vec{P}\cdot\hat{n}$

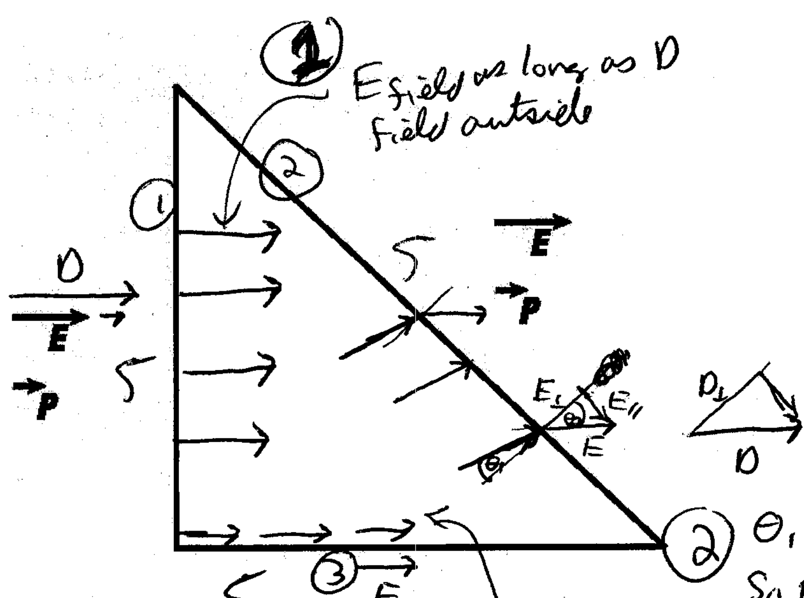
surfaces +1 1 $\vec{P}\cdot\hat{n} = P$ sign is "+"

+1 2 $\vec{P}\cdot\hat{n} = -P\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}P$ sign is "−" } +1



+1 3 $\vec{P}\cdot\hat{n} = 0$

b) (15pts) On the figure below, draw the electric field vectors (at regular intervals on all surfaces) just inside the cavity surface. Make sure you pay attention to the angle, direction and length (i.e. relative field strength) of the field vectors, in comparison to the electric field in the dielectric bulk.



$D = E + 4\pi P$

$D_\perp$ cont.

$E_\parallel$ cont.

$D_{outside} = \epsilon E$

$D_{inside} = E$

$\theta_1 < \theta_2$

So E field closer to the Normal

reasoning +3

drawing +2

③ E field the same magnitude of E field outside

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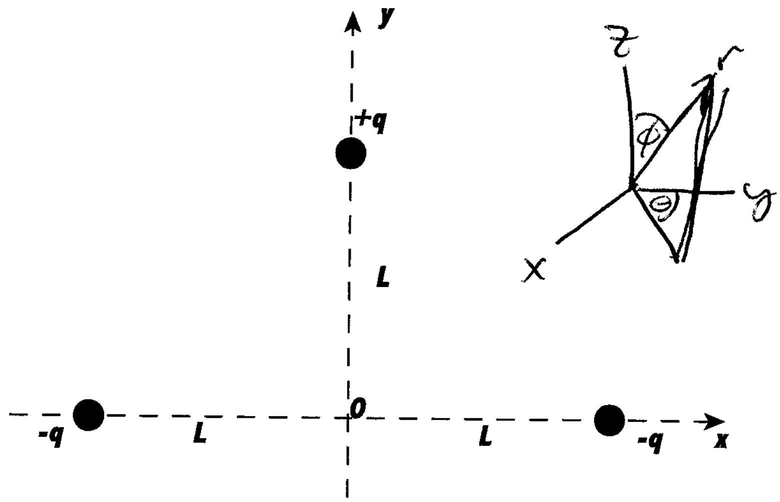
Question 2: Multipole Potentials (20 pts)

Suppose three point charges are arranged in a triangle, as shown below. Each charge is a distance L from the origin.



Find the first three multipole moments in the cartesian coordinates shown and their corresponding potentials ~~in spherical coordinates~~. Remember to take advantage of any symmetries etc to simplify your calculations.

$$\boxed{Q = \sum_\alpha q_\alpha = -q \qquad \Phi^{(1)} = \frac{-q}{r}}$$

+5

$$\vec{p} = \sum_\alpha q_\alpha \vec{r}_\alpha = \left[(-q)(-L) + (-q)(+L)\right]\hat{x} + qL\hat{y} = qL\hat{y}$$

$$\Phi^{(2)} = \frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{qL\cos\theta\sin\phi}{r^2}$$

+5

$$Q_{ij} = \sum_\alpha q_\alpha\left(3x_{i\alpha}x_{j\alpha} - r_\alpha^2\delta_{ij}\right)$$

$$Q_{11} = -q\left[3(-L)^2 - L^2\right] + -q\left(3L^2 - L^2\right) + q\left(-L^2\right) = -2qL^2 - 2qL^2 - qL^2 = -5qL^2$$

$$Q_{22} = -q\left(-L^2\right) - q\left(-L^2\right) + q\,3\left(L^2 - L^2\right) = 4qL^2$$

$$Q_{33} = -q\left(-L^2\right) - q\left(-L^2\right) + q\left(L^2\right) = qL^2$$

$$Q = \begin{pmatrix} -5qL^2 & 0 & 0 \\ 0 & 4qL^2 & 0 \\ 0 & 0 & qL^2 \end{pmatrix}$$

+5

traceless, symmetric

$$Q_{i\neq j} = 0$$

$$\Phi^{(4)} = \frac{1}{6}\sum_{ij} Q_{ij}\left(\frac{3x_ix_j - r^2\delta_{ij}}{r^5}\right)$$

$x = r\sin\phi\sin\theta$
$y = r\sin\phi\cos\theta$
$z = r\cos\phi$

$$\Phi^4 = \frac{1}{6}\left[\left(-5qL^2\right)\right]\frac{3\sin^2\theta\sin^2\phi - 1}{r^3} + \frac{4qL^2}{6}\,\frac{3\cos^2\theta\sin^2\phi - 1}{r^3} + \frac{qL^2}{6}\,\frac{\cos^2\phi - 1}{r^3}$$

$$\boxed{\Phi^4 = \frac{-15qL^2}{6r^3}\sin^2\theta\sin^2\phi + \frac{12}{6}\frac{qL^2}{r^3}\cos^2\theta\sin^2\phi + \frac{qL^2}{6r^3}\cos^2\phi}$$

+5