

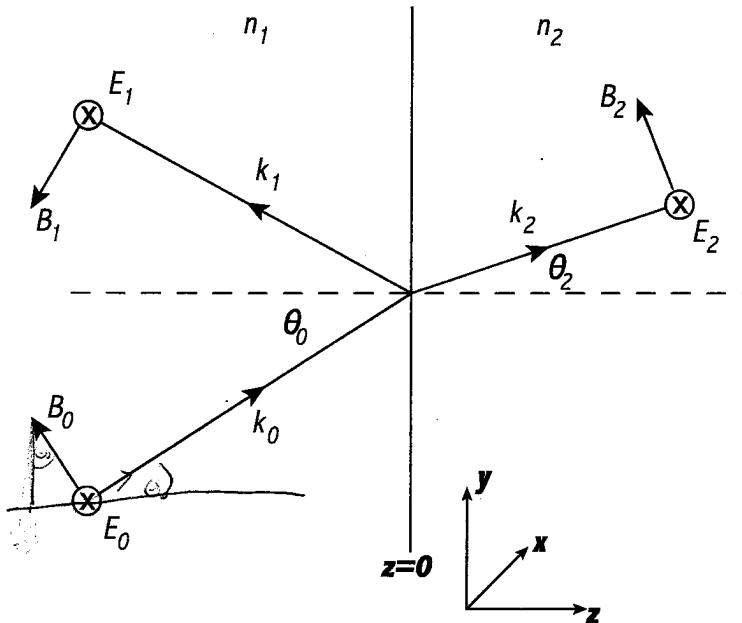
Quiz 2

Question 1: Reflection Pressure

An EM plane wave is incident upon a planar dielectric interface located at $z = 0$ in the coordinates shown below. The refractive indices of the two dielectrics are n_1 and n_2 respectively, and the plane wave is polarized perpendicularly to the plane of incidence. As we derived in lecture, the appropriate Fresnel equations are

$$E_1 = E_0 \frac{n_1 \cos \theta_0 - n_2 \cos \theta_2}{n_1 \cos \theta_0 + n_2 \cos \theta_2} \quad E_2 = E_0 \frac{2n_1 \cos \theta_0}{n_1 \cos \theta_0 + n_2 \cos \theta_2}. \quad (1)$$

Suppose that $n_2/n_1 \rightarrow \infty$, that is, the refractive index n_2 becomes very large.



- a) Write down the electric and magnetic field $\mathbf{E}_0(x, t)$ and $\mathbf{B}_0(x, t)$ for the incident radiation. Be sure to include the spatial and time dependences, as well as the polarization of the fields.

$$\begin{aligned} \vec{E}_0(x, t) &= E_0^0 e^{i(k_0 z - \omega t)} \hat{e}_x = E_0^0 e^{i(k_0 \cos \theta_0 z + k_0 \sin \theta_0 y - \omega t)} \hat{e}_x \\ \vec{B}_0(x, t) &= B_0^0 (\cos \theta_0 \hat{e}_y - \sin \theta_0 \hat{e}_z) e^{i(k_0 \cos \theta_0 z + k_0 \sin \theta_0 y - \omega t)} \\ |\vec{B}_0| &= n_1 \hat{e}_x \times \vec{E}_0 = n_1 E_0^0 (\cos \theta_0 \hat{e}_y - \sin \theta_0 \hat{e}_z) e^{i(k_0 \cos \theta_0 z + k_0 \sin \theta_0 y - \omega t)} \end{aligned}$$

- b) Write down $\mathbf{E}_1(x, t)$ and $\mathbf{E}_2(x, t)$ in this limit, and explain why this limit corresponds to perfect reflection of the incident radiation.

$$\lim_{n_2 \rightarrow \infty} \vec{E}_1 = -\vec{E}_0^0 \quad \vec{E}_2 = 0$$

$$(x3) \quad \vec{E}_1(x, t) = -E_0^0 e^{i(k_1 \cos \theta_0 z + k_1 \sin \theta_0 y - \omega t)} \hat{e}_x \quad (k_1 = |k_0|)$$

$$(x5) \quad (x1) \quad \vec{E}_2(x, t) = 0 \quad (+!) \text{ no wave is trans. reflected wave has changed its phase by } \pi$$

- c) Find $\mathbf{B}_1(x, t)$, and hence show that the total electric and magnetic fields $\mathbf{E}(x, t)$ and $\mathbf{B}(x, t)$ on the left side of the interface at $z = 0$ is

$$\mathbf{E}(y, t) = 0, \quad \mathbf{B}(y, t) = 2E_0 n_1 \cos \theta_0 e^{i(k_0 \sin \theta_0 y - \omega t)} \hat{e}_y. \quad (2)$$

$$\begin{aligned} \vec{B}_1(x, t) &= n_1 (-\cos \theta_0 \hat{e}_y + \sin \theta_0 \hat{e}_z) \times \vec{E}_1(x, t) \\ (x3) \quad &= n_1 E_0^0 e^{i(-k_1 \cos \theta_0 z + k_1 \sin \theta_0 y - \omega t)} (\cos \theta_0 \hat{e}_y + \sin \theta_0 \hat{e}_z) \end{aligned}$$

$$(x) \quad \vec{E}(x, t) = \vec{E}_0 + \vec{E}_1 = \vec{E}_0 + E_0^0 e^{i(k_0 \sin \theta_0 y - \omega t)} \hat{e}_x - E_0^0 e^{i(k_1 \sin \theta_0 y - \omega t)} \hat{e}_x = 0$$

$$(x) \quad \vec{B}(x, t) = \vec{B}_0 + \vec{B}_1 = 2F_0 n_1 \cos \theta_0 e^{i(k_0 \sin \theta_0 y - \omega t)} \hat{e}_y \quad \checkmark$$

Sin's Convol. Cos's odd.

$$\text{At } z=0 \quad e^{i k_0 \cos \theta_0 z} = 1$$

- d) Using your results and the Maxwell stress tensor, find the time-averaged pressure produced on the interface by the reflected radiation.

$$(+) \quad \langle T_{ij} \rangle = \frac{1}{4\pi n} [E_i E_j^* + B_i B_j^* - \frac{1}{2} \delta_{ij} (|E|^2 + |B|^2)]$$

$$\vec{E}(x,t) = 0 \quad \vec{B}(x,t) = 2n_1 E_0 \cos \theta_0 e^{i(k \sin \theta_0 y - wt)} \hat{e}_y$$

$$(+) \quad |B|^2 = 4n_1^2 E_0^2 \cos^2 \theta_0 \quad \cancel{\text{cancel}}$$

$$(+) \quad \langle T \rangle = \frac{n_1^2 E_0^2 \cos^2 \theta_0}{4\pi} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(+) \quad \langle P \rangle = - \hat{e}_z \cdot \underline{T} \cdot \hat{e}_z = \boxed{\frac{n_1^2 E_0^2 \cos^2 \theta_0}{4\pi}}$$

Formulae

$\sigma_b = \mathbf{n} \cdot \mathbf{P}$	$\rho_b = -\nabla \cdot \mathbf{P}$	$\frac{\mathbf{K}_b}{c} = -\mathbf{n} \times \mathbf{M}$	$\frac{\mathbf{J}_b}{c} = \nabla \times \mathbf{M}$
$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$	$\mathbf{P} = \chi_e \mathbf{E}$	$\mathbf{D} = \epsilon \mathbf{E}$	
$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$	$\mathbf{M} = \chi_m \mathbf{H}$	$\mathbf{B} = \mu \mathbf{H}$	
$\Phi(\mathbf{r}) = \int_V dv' \frac{\rho(\mathbf{r}')}{ \mathbf{r} - \mathbf{r}' }$	$\mathbf{E} = -\nabla\Phi$	$\mathbf{A}(\mathbf{r}) = \int_V dv' \frac{\mathbf{J}(\mathbf{r}')}{ \mathbf{r} - \mathbf{r}' }$	$\mathbf{B} = \nabla \times \mathbf{A}$
$Q = \sum_{\alpha} q_{\alpha}$	$\Phi^{(1)} = \frac{Q}{r}$	$\mathbf{p} = \sum_{\alpha} q_{\alpha} \mathbf{r}'_{\alpha}$	$\Phi^{(2)} = \frac{\mathbf{p} \cdot \mathbf{r}}{r^3}$
$\langle f(t)g(t) \rangle = \frac{1}{2}f_0g_0^*$	$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H}$	$d\mathbf{F} = \mathbf{T} \cdot d\mathbf{A}$	$g_{\text{field}} = \frac{1}{c^2} \mathbf{S}$
	$\mathbf{B} = n \mathbf{e}_k \times \mathbf{E}$	$\mathbf{E} = -\frac{1}{n} \mathbf{e}_k \times \mathbf{B}$	
$T_{ij} = \frac{1}{4\pi} \left[E_i E_j + B_i B_j - \frac{1}{2} \delta_{ij} (E^2 + B^2) \right]$		$\nabla \cdot \mathbf{S} = \frac{\partial g_{\text{field}}}{\partial t} + \frac{\partial g_{\text{matter}}}{\partial t}$	